# Sequential Contests\*

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#### Abstract

We compare expected rent-seeking expenditures and efficiency of simultaneous versus sequential rent-seeking contests. We find that when two risk neutral *ex ante* identical agents are competing, sequential contests are *ex ante* Pareto superior to simultaneous contests. We then endogenize the timing decision of rent-seeking expenditures and show that with ex ante identical contestants, all subgame perfect equilibria of this game are sequential contests.

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# 1 Introduction

In modeling rent-seeking competition, the standard framework is that first proposed by Tullock (1980) consisting of two or more rent-seekers *simultaneously* determining their rent-seeking expenditures with the "prize" then being awarded on the basis of some contest success function where success for player i is increasing in his rent-seeking expenditures. The main justification for this modeling assumption is that each contestant would have an incentive to be the first mover in a sequential contest; hence, any exogenous specification of the order of moves would be artificial. As a result, models of simultaneous rent-seeking expenditures have become standard in the rent-seeking framework (see Nitzan, 1994 for a survey).

Yet, in many contest settings, effort expenditures by contestants occur sequentially rather than simultaneously. In litigation, the development and presentation of evidence occurs sequentially. In politics, legislative agendasetting on different issues competing for limited funds induces sequential effort expenditures. The ordering of presidential conventions is conducted sequentially. Interestingly, the timing of the conventions from 1948 to present seems to be governed by the informal rule that the party of the sitting president always has its convention closer to the general election. This rule holds regardless of whether the sitting president is up for reelection or not and regardless of whether there is dissent over who should be elected (see Table 1).

[Table 1 about here]

In a number of industries, including the audit industry, it is customary (although not contractually obligated) for clients to give the incumbent firm the right to make a final offer for an engagement after learning about the offers of its rivals. A similar final offer scheme (sometimes contractually enforced, sometimes not) appears in making offers to free agents in a variety of professional sports. A more formal example of this sequential scheme arises in legal proceedings where it is customary for the plaintiff to present evidence prior to the defense.

Common to all of these examples is that the order in which rent-seeking expenditures are undertaken is determined prior to the realization of the relative valuations to each of the parties over the prize for which they are competing. In the case of presidential conventions, the timing must be committed to years in advance when, presumably each party has only a rough idea of the valuations of itself and its rival. However, at the time the rent-seeking expenditures themselves are undertaken by each party, the standard assumption that the valuations of the parties are commonly known appears reasonable. In view of this distinction between the information available at the time that the timing decision is undertaken versus when the actual rent-seeking expenditures take place, we focus on evaluating the welfare properties of simultaneous versus sequential contests from an ex ante perspective.

Specifically, we examine these properties in an *ex ante* symmetric environment consisting of two risk-neutral competitors competing in a contest. Despite the fact that the competitors are *ex ante* symmetric, we allow for the possibility of asymmetric contests to arise by allowing for both high and low value realizations to occur independently for each of the competing parties. The main result is that sequential contests *ex ante* Pareto dominate simultaneous contests. While this suggests that it would be a simple matter to sustain sequential contests as equilibria in finite or infinitely repeated games, we extend the model to obtain a stronger result. In a one-shot framework in which the timing decision is made simultaneously, all subgame perfect equilibria entail sequential competition. Thus, we offer an equilibrium model with ex ante symmetric agents where sequential contests arise endogenously absent any repeated game considerations.

Intuitively, sequential contests Pareto dominate simultaneous contests by creating a larger gap in the rent-seeking expenditures of the two contestants when their values differ. Since the discriminatory power of the auction in awarding the prize to the higher valued contestant is increasing in the gap between their expenditures, the sequential contest attains greater efficiency than does the simultaneous contest. At the same time, sequential contests create opportunities for preemptive bidding on the part of the first-mover. This has the effect of increasing the average amount of rent-seeking expenditures relative to the sequential contests. Thus, there is a tension between efficiency gains versus increasing rent-seeking expenditures in the sequential contest; however, the net effect is that the efficiency gains more than compensate for the costs of more aggressive bidding thus resulting in utility gains for the contestants. Moreover, including the utility of an agent receiving the proceeds of the contest yields our main result that sequential contests Pareto dominate simultaneous contests.

The paper is most closely related to Leininger (1993) who also examines the timing of rent-seeking expenditures to show that sequential competition might arise endogenously. Our paper differs from Leininger along several dimensions. First, Leininger models the timing decision as occurring following the realization of valuations for each of the contestants. In his model, endogenous sequential play arises (uniquely) only when contestants have asymmetric valuations. In the case where bidders are symmetric, either sequential or simultaneous play can arise as an equilibrium. In contrast, our model assumes that the timing decision is determined "behind the veil"; that is prior to the realization of specific valuations, and our bidders are ex ante symmetric. Moreover, we obtain the result that all subgame perfect equilibria of the timing game entail only sequential competition. In our model, the prospect of asymmetric value realizations breaks the indifference of bidders between simultaneous and symmetric contests in Leininger's model.

The remainder of the paper proceeds as follows: In section 2, we present the basic model and characterize the unique equilibrium under each contest form. Section 3 compares the welfare properties of the two contest forms. Section 4 then endogenizes the choice of simultaneous versus sequential contests Finally, section 5 concludes. Proofs of all propositions are contained in an appendix.

# 2 Preliminaries

Two risk neutral and ex ante identical agents are competing for some object. The object may be thought of as a piece of legislation, a contract for services, a telecoms license, and so forth. The value of the object to agent i (i = 1, 2) is  $V_i$ , which takes on the high value  $V^H$  with probability p and the low value  $V^L$  with probability 1 - p, and where  $V^H > V^L > 0$ . The realization of each agent's valuation for the reward is independent. In the event that an agent does not receive the object, she earns zero.

Agents compete for the object by making irreversible effort outlays. The effort of agent *i* is denoted  $x_i$ . The valuation that each agent places on the reward is commonly known prior to making effort outlays. Given the effort outlays, agent *i* receives the object with probability  $\lambda_i = \frac{x_i}{x_i + x_j}$ . Thus, the expected utility of agent *i* when the object is worth  $V_i$  and the effort outlay of the other agent is  $x_j$  is given by:

$$EU_i = \frac{x_i}{x_i + x_j} V_i - x_i. \tag{1}$$

The analysis in this paper compares two different institutions used to resolve the contest—simultaneous and sequential effort expenditures. Before proceeding, it is useful to offer some justification for this modeling framework. First, the simultaneous form of this basic framework has been widely used to analyze a wide variety of imperfectly discriminating contests and auctions across a number of settings including lobbying competition, technology races, and defence expenditures (See Nitzan (1994) for further examples.) Thus, it is useful to understand the implications of the modeling decision of simultaneous expenditures. Second, although the contest success function is assumed in this model, Skaperdas (1996) and Clark and Riis (1998) offer axiomatic foundations for contest success functions having this form. Third, even though we are taking the timing of the contest to be exogenous at this point, in section 4 this decision is endogenized—and sequential contests arise endogenously. Finally, the effect of increasing the difference in expenditures between the low and high valued agent, which occurs in this framework and drives our welfare results, will be present in some form in most imperfectly discriminating contests. The specification in equation (1) provides a simple and tractable way to illustrate that the efficiency gains in sequential contests are then allocated among all the parties to create the possibility of a Pareto improvement compared to the simultaneous contest.

We begin by deriving equilibrium contest outlays under each institution.

Proofs for all propositions are contained in an appendix. First, consider the case where outlays are made simultaneously. In this case, we have:

**Proposition 1** The unique Nash equilibrium of the simultaneous contest is

$$x_i = V_i V_j \frac{V_i}{\left(V_i + V_j\right)^2}$$

We now characterize the unique equilibrium when contest outlays are made sequentially. **Proposition 2** The unique subgame perfect equilibrium of the sequential contest is

$$x_{1} = \frac{V_{1}}{2} \left( \frac{V_{1}}{2V_{2}} \right) \quad x_{2} = \frac{V_{1}}{2} \left( 1 - \frac{V_{1}}{2V_{2}} \right) \quad if \ V_{1} \le 2V_{2}$$
$$x_{1} = V_{2} \qquad x_{2} = 0 \qquad otherwise$$

Figure 1 illustrates how the equilibrium outcomes of sequential contests differ from simultaneous contests for the case where  $V_1 = V^H$  and  $V_2 = V^L$  and  $V_1 \leq 2V_2$ . The best response functions for each agent are denoted  $BR_1(x_2)$  and  $BR_2(x_1)$ , respectively. In the simultaneous contest, point Athen represents equilibrium rent-seeking expenditures.

#### [Figure 1 here]

In a sequential contest, the subgame perfect equilibrium shifts to point B where agent 1's indifference curve is tangent to  $BR_2(x_1)$ . Thus, agent 1 commits to a more aggressive strategy knowing that this will induce a substantial reduction in rent-seeking expenditures by agent 2. In contrast, if agent 2 moves first, then the sequential contest shifts the equilibrium to point C, where agent 2's indifference curve is tangent to  $BR_1(x_2)$ . Here, agent 2 commits to economize on rent-seeking expenditures knowing that agent 1 will follow suit.

### **3** Welfare Analysis

In this section we compare the welfare properties of simultaneous and sequential contests. The main result of the paper is to show that under a variety of welfare criteria, the sequential contest is superior to the simultaneous contest. First, consider the case where the outlays represent pure transfers from the competing agents to the organization running the contest. This would be the case when the contest was an auction. In this case, a relevant welfare criterion is allocative efficiency—the probability that the contest allocates the object to the agent valuing it most highly.

**Proposition 3** The sequential contest increases allocative efficiency over the simultaneous contest.

Intuitively, sequential contests increase the gap between expenditures of high value agents and those with low value both when the first mover is high valued as well as when she is low valued. As a consequence, the probability that the contest awards the prize to the high valued agent increases in the sequential case. This may be seen in Figure 1. In both cases, rent-seeking expenditures in the sequential contest are more unequal (as measured by the distance from the 45 degree line); however the gap between high and low value expenditures is not independent of the ordering of moves. In particular, the gap is always larger when the first mover has a high value. Recognizing this, it then follows that

**Proposition 4** Maximal contest efficiency is attained in a sequential contest when the first mover has the (weakly) higher value for the prize.

Again, viewing the outlays as pure transfers from the agents to the organization running the contest, a natural question is which contest form raises more revenues. The next proposition shows that sequential contests outperform simultaneous contests in terms of expected revenues.

**Proposition 5** The sequential contest leads to higher total effort outlays than the simultaneous contest.

The intuition for this result is as follows: The sequential contest offers the first contestant the ability to commit to a publicly observed level of expenditures. In the case where the valuations of both parties are identical, this commitment does nothing to change the rent-seeking expenditures of either party since for small changes in expenditures in the neighborhood of the simultaneous equilibrium this induces no change in the expenditures of the second contestant. Notice that this is exactly the reasoning underlying the Nash equilibrium in the simultaneous game; hence the power of commitment does not affect equilibrium expenditures.

In contrast, when the first contestant has a high value and the second has a low value, then higher expenditures by agent 1 are met by lower expenditures by agent 2. As a consequence, agent 1 can credibly commit to increase his expenditures knowing that this will be profitable in increased probability of winning owing to agent 2's reduction in expenditures. In cases of extremely unequal values, this reasoning leads to a corner solution where agent 1 spends the full value of the prize to agent 2 in order to ensure that agent 2 does not participate at all in the contest. The net effect of this is to increase effort outlays relative to the simultaneous contest.

Finally, in the case where agent 1 has a low value and agent 2 has a high value, agent 1 can afford to reduce her effort expenditures knowing that 2 will follow suit and likewise reduce. In this case, the power to commit to a lower level of expenditures enables that agent to credibly reduce the overall "aggressiveness" of the contest. This decreases the marginal gain in the probability of winning at higher expenditure levels thus inducing agent 2 to scale back expenditures. Despite this reduction in expenditures relative to the simultaneous game for his pair of realizations, the revenue effects are outweighed by the increased effort expenditures occurring when agent 1 has a high value.

### [Figure 2 here]

This may be seen in Figure 2, which illustrates the change in aggregate rent-seeking expenditures in the sequential contest. Drawing an isoexpenditures line through point A, it is apparent that point B results in increased rent-seeking expenditures whereas point C reduces overall expenditures. Ex ante, points B and C are equally likely; thus, the expected rent-seeking expenditures in the sequential contest are given by point D, which is midway between points B and C. Notice, however, the point D also lies above the iso-expenditure line; hence the sequential contest increases expected effort expenditures.

Finally, we turn to the *ex ante* expected utility of the agents competing in the contest. Once again, the sequential contest is superior.

**Proposition 6** The sequential contest leads to higher ex ante expected utility than the simultaneous contest.

Compared to the simultaneous game, the sequential rent-seeking game has two effects. In the case in which the valuations of the two bidders differ, the sequential contest leads to greater competition between the agents as the first agent tries to use its initial move to preempt (in the case of high value) or calm down (in the case of low value) effort expenditures by the second group. This has the effect of increasing expected effort expenditures *ex ante*. At the same time, the sequential contests also leads to more uneven expenditures between the two parties when their values differ. That is the gap between the expenditures of the high and low valued agents increase in the sequential game. As the difference in expenditures grows more unequal, this increases the probability that the prize will be awarded to the party valuing it more highly. As a consequence, expected contest efficiency increases relative to simultaneous contests. The increased surplus owing to this efficiency gain, allows both of the rent-seeking parties to enjoy higher expected surplus under this form despite the fact that the overall rent-seeking expenditures have increased. Thus, the sequential contest Pareto dominates the simultaneous contest. Viewed in this light, it then follows that if parties participating in such contests can then commit to a contest form, they will naturally choose the sequential contest form. Alternatively, if the parties are repeatedly competing against one another in contests, it seems reasonable that they might coordinate on the sequential extensive form.<sup>1</sup> We summarize these observations as

**Proposition 7** Sequential contests ex ante Pareto dominate simultaneous contests.

Wasteful Rent-seeking In many circumstances, rent-seeking expenditures are not properly thought of as welfare neutral transfers from the contestants to the person conducting the contest. Instead, part or all of these expenditures might represent purely wasteful activities. We study the polar case where rent-seeking expenditures are purely wasteful and compare simultaneous to sequential contests.

The results of Proposition 6 are helpful in this regard. From the perspective of the agents competing in the contest, whether rent-seeking expenditures represent transfers or pure waste is irrelevant to their expected utility calculation. Proposition 6 shows that each agent's ex ante expected utility is higher under sequential contests than under simultaneous contests. Thus, despite the fact that sequential rent-seeking contests generate greater expected total effort outlays, which, in this context are purely wasteful, the social cost of these additional expenditures is more than compensated for by the gains in allocative efficiency arising from the sequential form of the contest.

### 4 Endogenous Timing in Contests

We now extend the basic model to allow participants to choose the timing of their effort expenditures prior to the realization of valuations. Formally, each participant chooses whether her rent-seeking expenditures will take place in one of two periods (period 1 or period 2). Period 2 occurs following period 1 and all rent-seeking expenditures undertaken in period 1 are publicly observed by the start of period 2. The periods are close enough in time such that discounting is not a factor for any of the participants of the contest nor for any individuals receiving the rent-seeking expenditures.

Thus, the extensive form of the game is as follows: in period 0, both participants simultaneously choose whether to commit rent-seeking expenditures in period 1 or period 2. At the conclusion of period 0, the valuations are realized and publicly revealed. In period 1, all participants who committed to making rent-seeking expenditures in this period simultaneously choose expenditure levels. In period 2, all period 1 expenditures are publicly revealed and participants committing to period 2 expenditures simultaneously choose expenditure levels. At the conclusion of period 2, the contest is resolved and payoffs are realized. Throughout, we restrict attention to subgame perfect equilibria of the game. Using the results in the previous section, the rent-seeking expenditures for each possible realization of the dynamic game have already been characterized. It then remains to analyze the outcome of the period 0 game. The expected utility of a participant in the event that the rent seeking expenditures are committed to in the same period as her competitor is

$$EU = (1-p)^{2} \frac{1}{2} V^{L} + p^{2} \frac{1}{2} V^{H} + (1-p) p \left(\frac{V^{L}}{V^{L} + V^{H}}\right) V^{L} + p (1-p) \left(\frac{V^{H}}{V^{L} + V^{H}}\right) V^{H} - p^{2} \frac{V^{H}}{4} - (1-p)^{2} \frac{V^{L}}{4} - p (1-p) \left(\frac{(V^{H})^{2} V^{L} + V^{H} (V^{L})^{2}}{(V^{L} + V^{H})^{2}}\right)$$

In the event that the participant is moving first in a sequential game and  $V^H \leq 2 V^L \text{ we have }$ 

$$EU_{1} = (1-p)^{2} \frac{1}{2} V^{L} + p^{2} \frac{1}{2} V^{H} + (1-p) p \left(\frac{V^{H}}{2V^{L}}\right) V^{H} + p (1-p) \left(\frac{V^{L}}{2V^{H}}\right) V^{L} - p^{2} \frac{V^{H}}{4} - (1-p)^{2} \frac{V^{L}}{4} - (1-p) p \left(\frac{V^{H}}{2} \left(\frac{V^{H}}{2V^{L}}\right) + \frac{V^{L}}{2} \left(\frac{V^{L}}{2V^{H}}\right)\right)$$

Finally, in the event that the participant is moving second in a sequential

game and  $V^H \leq 2V^L$  we have

$$EU_{2} = (1-p)^{2} \frac{1}{2} V^{L} + p^{2} \frac{1}{2} V^{H} + (1-p) p \left(1 - \frac{V^{H}}{2V^{L}}\right) V^{L} + p (1-p) \left(1 - \frac{V^{L}}{2V^{H}}\right) V^{H} - p^{2} \frac{V^{H}}{4} - (1-p)^{2} \frac{V^{L}}{4} - (1-p) p \left(\frac{V^{H}}{2} \left(1 - \frac{V^{H}}{2V^{L}}\right) + \frac{V^{L}}{2} \left(1 - \frac{V^{L}}{2V^{H}}\right)\right)$$

It is a simple matter to verify that  $EU_1 = EU_2$ , so, from an ex ante perspective, there is no particular advantage to going first in a sequential contest. This is perhaps somewhat surprising given the emphasis placed on first-mover advantages in the existing literature. We now compare the expected utility in the sequential contest to that of a simultaneous contest. Differencing

$$EU_{1} - EU = (1 - p) p \left( \frac{1}{4} \left( \left( V^{L} \right)^{2} - V^{H} V^{L} + \left( V^{H} \right)^{2} \right) \frac{\left( V^{L} - V^{H} \right)^{2}}{V^{H} V^{L} \left( V^{L} + V^{H} \right)} \right)$$
  
> 0

It is routine to verify that the same result holds for the case where  $V^H > 2V^L$ . It then follows that:

Proposition 8 In all subgame perfect equilibria of the two stage game, the

participants select sequential contests.

Thus, we have shown that no repeated game arguments are necessary to sustain sequential competition in rent seeking contests. The only difficulty would seem to be coordinating on an equilibrium pair. In practice, rules such as those in which the incumbent goes last, as in pricing of audit engagements or determining order in presidential conventions might offer a mechanism to ensure coordination on a particular sequential outcome.<sup>2</sup>

# 5 Conclusion

In this paper, we have endogenized the timing of rent-seeking expenditures and analyzed the *ex ante* welfare properties of sequential versus simultaneous contests to reconcile the many instances of sequential competition appearing in practice with the presumption of all-out simultaneous competition prevalent in the existing literature. Specifically, we established that with *ex ante* identical individuals, sequential contests Pareto dominate simultaneous contests (even from the perspective of the organization receiving the rent-seeking expenditures). Thus, one may readily suggest repeated game arguments for their implementation. However, even absent any repeated game considerations, by modeling the timing of rent-seeking expenditures as an ex ante commitment on the part of each of the players, we obtain the result that all subgame perfect equilibria of the timing game result in sequential contests. This is one possible explanation for why even short-run players (such as presidential candidates) do not resort to all-out simultaneous competition in high stakes rent-seeking games.

# **A** Proofs of Propositions

**Proof of Proposition 1** Differentiating equation (1) with respect to  $x_i$  for each agent yields a system of simultaneous equations satisfying

$$x_j V_i = (x_i + x_j)^2$$
. (2)

This implies

$$x_j V_i = x_i V_j.$$

Substituting this back into equation (2) yields the solution

$$x_i = V_i V_j \frac{V_i}{\left(V_i + V_j\right)^2}$$

The proof of uniqueness is standard (see Nitzan (1994) for instance.)

**Proof of Proposition 2** Given  $x_1$  and the realization  $(V_1, V_2)$ , agent 2's optimization is to choose  $x_2$  to maximize equation (1). Differentiating with respect to  $x_2$ , setting the result equal to zero, and solving for  $x_2$  yields 2's

best response function

$$x_2(x_1) = \sqrt{x_1 V_2} - x_1. \tag{3}$$

Now agent 1 chooses  $x_1$  to maximize equation (1) given that agent 2 will best respond to 1's choice. Differentiating with respect to  $x_1$  and setting the result equal to zero yields:

$$\left(x_2 - x_1 \frac{dx_2}{dx_1}\right) V_1 = \left(x_1 + x_2\right)^2.$$

Substituting for  $x_2$  and  $\frac{dx_2}{dx_1}$  using equation (3) and its derivative, respectively, yields

$$\left(\sqrt{x_1V_2} - x_1 - x_1\left(\frac{\sqrt{V_2}}{2\sqrt{x_1}} - 1\right)\right)V_1 = \left(x_1 + \sqrt{x_1V_2} - x_1\right)^2.$$

Solving for  $x_1$  and accounting for non-negativity constraints of outlays yields the solution

$$x_{1} = \frac{V_{1}}{2} \left( \frac{V_{1}}{2V_{2}} \right) \quad x_{2} = \frac{V_{1}}{2} \left( 1 - \frac{V_{1}}{2V_{2}} \right) \quad \text{if } V_{1} \le 2V_{2}$$
$$x_{1} = V_{2} \qquad \qquad x_{2} = 0 \qquad \qquad \text{otherwise}$$

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Strict concavity of objective functions in each of the stage games guarantees uniqueness.

**Proof of Proposition 3** Using the equilibrium outlays in Proposition 1, the probability that the simultaneous contest misallocates the object is

$$\pi^{SIM} = 2p\left(1-p\right)\left(\frac{V^L}{V^H + V^L}\right).$$

Calculating in a manner analogous to the simultaneous case leads to the probability that the prize will be misallocated given by:

$$\pi^{SEQ} = p \left( 1 - p \right) \left( 1 - \frac{V^H}{2V^L} + \frac{V^L}{2V^H} \right).$$

 $\text{if }V^{H}\leq 2V^{L}$ 

Differencing these two expressions

$$\pi^{SIM} - \pi^{SEQ} = p(1-p) \left( \frac{2V^L}{V^H + V^L} - 1 + \frac{V^H}{2V^L} - \frac{V^L}{2V^H} \right)$$
$$= p(1-p) \left( \frac{\left( \left( V^H \right)^2 + \left( V^L \right)^2 \right) \left( V^H - V^L \right)}{2V^H V^L \left( V^H + V^L \right)} \right)$$
$$> 0.$$

If  $V^{H}>2V^{L}, \mbox{then}$  the probability of misal location in a sequential contest is

$$\pi^{SEQ} = (1-p) p\left(\frac{V^L}{2V^H}\right).$$

Again, differencing the misallocation probabilities under simultaneous versus sequential contests, we have

$$\begin{aligned} \pi^{SIM} - \pi^{SEQ} &= p \left( 1 - p \right) \left( \frac{2V^L}{V^H + V^L} \right) - \frac{V^L}{2V^H} \\ &= \frac{p \left( 1 - p \right)}{2 \left( V^H + V^L \right) V^H} \left( 3V^H V^L - \left( V^L \right)^2 \right) \\ &> 0, \end{aligned}$$

and the result follows.

**Proof of Proposition 4** To see that the statement of the Proposition holds when  $V^H \leq 2V^L$ , first observe that

$$1 - \frac{V^H}{2V^L} < \frac{V^L}{2V^H}.$$

This is equivalent to

$$\frac{\left(V^H - V^L\right)^2}{2V^H V^L} > 0.$$

When  $V^H > 2V^L$ , the sequential contest leads to full allocative efficiency if the high valued agent is the first mover.

**Proof of Proposition 5** In the simultaneous contest, we may use Proposition 1 to compute the expected effort outlays. These are given by

$$R^{SIM} = p^2 \frac{V^H}{2} + 2p \left(1 - p\right) \frac{V^H V^L}{V^H + V^L} + \left(1 - p\right)^2 \frac{V^L}{2}.$$

Similarly, we may use Proposition 2 to compute expected outlays in the case of sequential contests when  $V^H \leq 2V^L$ . These are given by

$$R^{SEQ} = p \frac{V^H}{2} + (1-p) \frac{V^L}{2}.$$

Differencing the two expected expenditure outlay terms

$$\begin{split} R^{SEQ} - R^{SIM} &= p \frac{V^H}{2} + (1-p) \frac{V^L}{2} - p^2 \frac{V^H}{2} - 2p \left(1-p\right) \frac{V^H V^L}{V^H + V^L} - (1-p)^2 \frac{V^L}{2} \\ &= p \left(1-p\right) \left(\frac{\left(V^H - V^L\right)^2}{2 \left(V^H + V^L\right)}\right) \\ &> 0. \end{split}$$

When  $V^H > 2V^L$ , expected outlays in a sequential contest are

$$R^{SEQ} = p^2 \frac{V^H}{2} + (1-p)\left(\frac{V^L}{2}\right) + p(1-p)V_L.$$

Differencing revenues in sequential versus simultaneous contests in this case yields

$$R^{SEQ} - R^{SIM} = p (1-p) \left( \frac{V^L}{2} + V^L - 2 \frac{V^H V^L}{V^H + V^L} \right)$$
$$= \frac{p (1-p)}{2 (V^H + V^L)} \left( 3 (V^L)^2 - V^L V^H \right)$$
$$> \frac{p (1-p)}{2 (V^H + V^L)} \left( 3 (V^L)^2 - 2 (V^L)^2 \right)$$
$$> 0,$$

and the result follows.

**Proof of Proposition 6** The *ex ante* expected utility in a simultaneous contest is

$$W^{SIM} = p^2 \frac{V^H}{2} + (1-p)^2 \frac{V^L}{2} + 2p(1-p) \left(\frac{(V^H)^2 - V^H V^L + (V^L)^2}{V^H + V^L}\right).$$

When  $V^H \leq 2V^L$ , we may use Proposition 2 to compute the *ex ante* expected utility in the sequential contest. This is given by

$$W^{SEQ} = p^2 \frac{V^H}{2} + (1-p)^2 \frac{V^L}{2} + p(1-p) \left(\frac{V^H}{2V^L}V^H + \left(1 - \frac{V^H}{2V^L}\right)V^L - \frac{V^H}{2}\right) + (1-p) p\left(\frac{V^L}{2V^H}V^L + \left(1 - \frac{V^L}{2V^H}\right)V^H - \frac{V^L}{2}\right)$$

Differencing these expressions yields

$$\begin{split} W^{SEQ} - W^{SIM} &= p \left( 1 - p \right) \left( \frac{V^H}{2V^L} V^H + \left( 1 - \frac{V^H}{2V^L} \right) V^L - \frac{V^H}{2} \right) \\ &+ \left( 1 - p \right) p \left( \frac{V^L}{2V^H} V^L + \left( 1 - \frac{V^L}{2V^H} \right) V^H - \frac{V^L}{2} \right) \\ &- 2p \left( 1 - p \right) \left( \frac{\left( V^H \right)^2 - V^H V^L + \left( V^L \right)^2}{V^H + V^L} \right) \\ &= p \left( 1 - p \right) \left( \frac{\left( \left( V^H \right)^2 - V^H V^L + \left( V^L \right)^2 \right) \left( V^L - V^H \right)^2}{(V^H + V^L) V^H V^L} \right) \\ &> 0. \end{split}$$

When  $V^H > 2V^L$ , the *ex ante* expected utility in a sequential contest is

$$\begin{split} W^{SEQ} &= p^2 \frac{V^H}{2} + (1-p)^2 \frac{V^L}{2} + p \left(1-p\right) \left(V^H - V^L\right) \\ &+ \left(1-p\right) p \left(\frac{V^L}{2V^H} V^L + \left(1-\frac{V^L}{2V^H}\right) V^H - \frac{V^L}{2}\right). \end{split}$$

Differencing the *ex ante* expected utility under sequential and simultaneous contests yields

$$\begin{split} W^{SEQ} - W^{SIM} &= p \left( 1 - p \right) \left( V^H - V^L + \frac{V^L}{2V^H} V^L + \left( 1 - \frac{V^L}{2V^H} \right) V^H - \frac{V^L}{2} \right) \\ &- 2 \left( 1 - p \right) p \left( \frac{\left( V^H \right)^2 - V^H V^L + \left( V^L \right)^2 \right)}{V^H + V^L} \right) \\ &= \frac{p \left( 1 - p \right)}{2} \left( 4 \left( V^H \right)^2 - 7 V^H V^L + \left( V^L \right)^2 \right) \\ &= \frac{p \left( 1 - p \right)}{2} \left( V^H \left( 4 V^H - 7 V^L \right) + \left( V^L \right)^2 \right) \\ &> \frac{p \left( 1 - p \right)}{2} \left( V^H V^L + \left( V^L \right)^2 \right) \\ &> 0, \end{split}$$

and the result then follows.

### Endnotes

\* The author acknowledges the support of the National Science Foundation and the Hoover Institution. The author is grateful to an anonymous referee for helpful comments. Please address all correspondence to: John Morgan, Haas School of Business, Berkeley, CA 94720; E-mail: rjmorgan@haas.berkeley.edu

1. Some care is required here. We have in mind a situation in which the only repeated game strategies are choosing between sequential versus simultaneous extensive forms of the game for period t. Specifically, suppose that players simultaneously choose between the strategies {Seq, Sim}. In the event that either player chooses Sim then the contest is played simultaneously. Now, it is routine to verify that the sequential play may be sustained as an equilibrium to an infinitely repeated game.

2. Or, alternatively, a publicly observable randomizing device in a correlated equilibrium.

# References

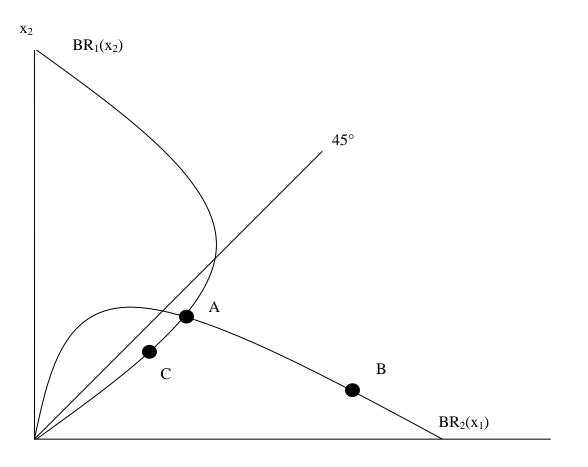
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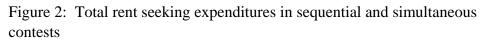
	Start Date			Last
Year	Democrat	Republican	Incumbent	Mover
1948	12-Jul	21-Jun	Democrat	Democrat
1952	21-Jul	7-Jul	Democrat	Democrat
1956	13-Aug	20-Aug	Republican	Republican
1960	11-Jul	25-Jul	Republican	Republican
1964	24-Aug	13-Jul	Democrat	Democrat
1968	26-Aug	5-Aug	Democrat	Democrat
1972	10-Jul	21-Aug	Republican	Republican
1976	12-Jul	16-Aug	Republican	Republican
1980	11-Aug	14-Jul	Democrat	Democrat
1984	16-Jul	20-Aug	Republican	Republican
1988	18-Jul	15-Aug	Republican	Republican
1992	13-Jul	17-Aug	Republican	Republican
1996	26-Aug	12-Aug	Democrat	Democrat
2000	13-Aug	30-Jul	Democrat	Democrat

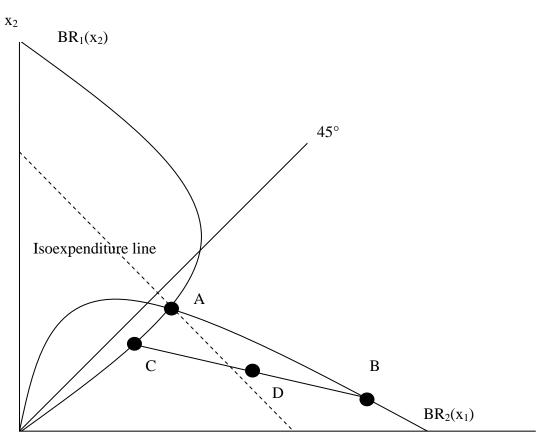
### Table 1: The order of presidential conventions 1948- present

Figure 1: Equilibrium outcomes in sequential and simultaneous contests



 $\mathbf{x}_1$ 





 $\mathbf{x}_1$